# On the Infinite Boundaries of Art and Language

Andrzej Lawn

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## 1 Introduction

When working with languages there are certain barriers in between one language and another, between syntax and semantics, between languages that are diametrically opposed to each other. Notwithstanding, barriers are barriers only from a certain perspective. Often one language can shed insight into another, unlocking latent nuances embedded within its structure. Such is the case with mathematics, language, and art.

## 2 Definitions

Definition: **Language** –

1. Any system of formalized symbols, signs, sounds, gestures, or the like used or conceived as a means of communicating a thought, idea, emotion, etc.
2. A body of words and the systems for their use.

Definition: **Mathematics** –

1. The systematic treatment of relationships between figures and forms, and relations between quantities expressed symbolically.
2. Procedures, operations, or properties relating to mathematics.

## 3 The Proof

Denote the set of all language, including its symbols and signs as $A_L$.

Denote the set of all mathematical language as the set containing all symbols and signs of mathematics as M.

Since M is simply a language contained in the set of all language $A_L$. Then M is contained in $A_L$ (ie $M \subseteq A_L$) or in other words M is a subset of $A_L$. Therefore M can be classified as another language contained in all languages $A_L$.

ASSUMPTION: **Art is a language** (ie Art is a system of symbols, signs, sounds, gestures or the like used or conceived as a means of communicating a thought, emotion, etc. A body of ideas and the systems for their presentation and or use).

Assuming Art is a language then we can model Art based upon an idiom. Similarly (as in the case of $M \subseteq A_L$) it can be shown that Art can be classified as a subset of the set of all language (ie if Art is a language than it must follow that art is contained in the set of all language $A_L$).

Let us construct the language of Art. Since art is a language and a language can be composed of a body of words then it suffices to define Art by finding all the words in the set $A_{rt}$ (ie the set containing the language of Art). Let us list the words in art as follows:

For simplicity assume that the language of Art uses the English alphabet (Note that a similar argument can be made for any other language). The English alphabet will be denoted by the set $E_{ALPHA}$.

$$E_{ALPHA} = \{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w,x,y,z,\ \}$$

(the last character in the set being used to represent a space " ")

OR

$$E_{ALPHA} = \{\text{the set of all expansions in the base 27}\}$$

ie

$$E_{ALPHA} = \{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26\}$$

Then we can list all possible words in the Art language $A_{rt}$ as follows:

$$\begin{aligned} word_1 &= a_1 \qquad \text{where } a_n \in E_{ALPHA}\ \forall\ n, \\ word_2 &= a_1a_2 \qquad \text{where } n \in \mathcal{N} \text{ (the set of natural numbers)} \\ word_3 &= a_1a_2a_3 \\ &\ \vdots \\ word_n &= a_1a_2a_3\ldots a_n\ldots \\ &\ \vdots \end{aligned}$$

In this representation, Art language $A_{rt}$ is reduced to a sequence of characters in the set $E_{ALPHA}$.

By applying *Cantor's Diagonal Argument*[§] to the listing of the set $A_{rt}$ then $A_{rt}$ is an uncountably infinite set.[1]

*Aside*: Although words generated from $A_{rt}$ may not contain words in the English language dictionary, in these infinite sequences all words in the English language dictionary are generated. We can easily take the intersections of the sets $A_{rt} \cap$ (the set of all words in the

[§] *Cantor's Diagonal Argument*, also called the diagonalization argument, the diagonal slash argument or the diagonal method, was published in 1891 by Georg Cantor as a proof that there are infinite sets which cannot be put into one-to-one correspondence with the infinite set of natural numbers. Such sets are now known as uncountable sets, and the size of infinite sets is now treated by the theory of cardinal numbers which Cantor began.

[1] We can define a function $f$ such that $f: A_{rt} \rightarrow S$, where S is the set of all infinite and finite sequences of elements of the form $(x_1, x_2, x_3, \ldots)$ where each element $x_i$ is either 0 or 1 (whereas S is an uncoutably infinite set proven in *Cantor's Diagonal Argument*[§]). This can be done by mapping the letters of the words in $A_{rt}$ as follows: for each $a_n$ in $word_n$ if $a_n$ is in the set $\{a,b,c,d,e,f,g,h,i,j,k,l,m\}$ then $a_n \rightarrow 0$ else $a_n \rightarrow 1$. $\therefore |A_{rt}| = |S|$.

English language), which is still an uncountably infinite set containing only words in the English language. Which can be represented as numbers in the base 1,000,000 if we assume that the number of words including scientific words in the English language is 1,000,000 words.

# 4 Implications

$\therefore$ $A_{rt}$ is an uncountably infinite set, then the language of art is infinite and cannot be set (ie there is no finite definition or set which can contain $A_{rt}$).

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